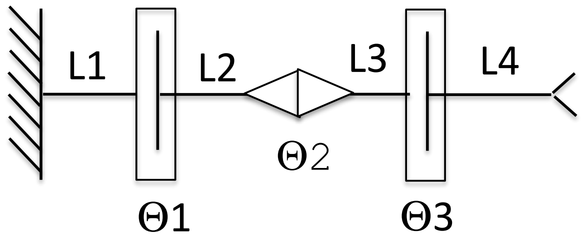
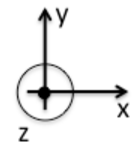
QUESTION 1 *Manipulator* **(2+3+3+3 = 11 points)**

Consider the following robot manipulator:





(a) Graphically sketch the work area (reachable area) of this manipulator. How would you best verbally describe this shape?

Graphical sketch:

Verbal description:

(b) Describe the Transformation from Base to Tool-Tip using *Rot*$_{axis}(\alpha)$ and *Trans(x,y,z)*.

T =

(c) Derive 3 Homogeneous Transformation for this manipulator as 4x4 matrices.

T = T1 * T2 * T3 (T1 from base until after $\Theta 1$, T2 until after $\Theta 2$, T3 until after $\Theta 3$)

T1 =

T2 =

T3 =

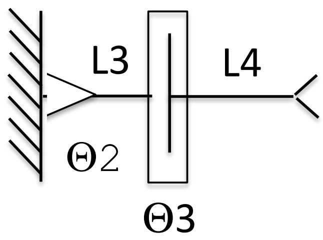
(d) For the special case that $\Theta 1=90^\circ$ and $\Theta 3=-90^\circ$, simplify to a single 4x4 transformation. What is the 3D-position of the tool-tip in this case for $\Theta 2=90^\circ$?

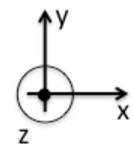
$T_{90,90}$ =

ToolTip =

QUESTION 2 ***Inverse Kinematics*** **(10 points)**

Simply the robot manipulator from before by fixing $\theta 1 = 0$, L1=0, L2=0, L3=1, L4=1

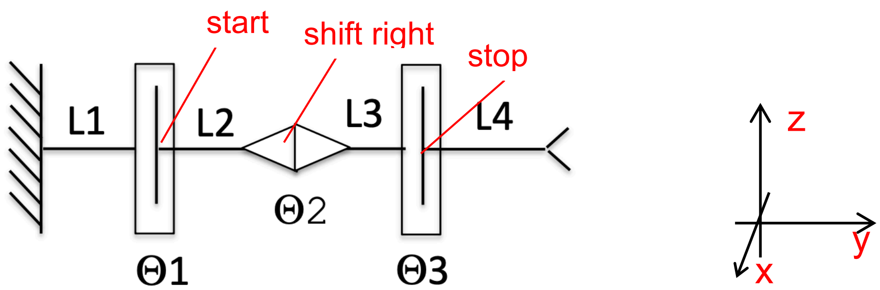




(a) Graphically sketch the work area (reachable area) of this manipulator. How would you best verbally describe this shape?

(b) Derive and $\theta 2$ and $\theta 3$ from given x, y, z coordinates of the end-effector.

QUESTION 3 ***Denavir-Hartenberg***
Consider the following robot manipulator:



(a) Complete the DH table for this given manipulator (ignore L1, L4). Remember that joints Θi should be along the z-axis, and links Li along the x or z-axis.

	Rotx	Transx	Transz	Rotz
i	α_{i-1}	a_{i-1}	d_i	θ_i
1				
2				
3				

(b) Write the manipulator forward kinematics as a sequence of DH Trans and Rot transformations.

T = ____

(c) Write ^0_1T, ^1_2T and ^2_3T (ignoring L4) as DH 4x4 matrices.

$$
{}^{i-1}_{\;\;i}T = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i \cdot \cos\alpha_{i-1} & \cos\theta_i \cdot \cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1} \cdot d_i \\ \sin\theta_i \cdot \sin\alpha_{i-1} & \cos\theta_i \cdot \sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1} \cdot d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

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^0_1T =

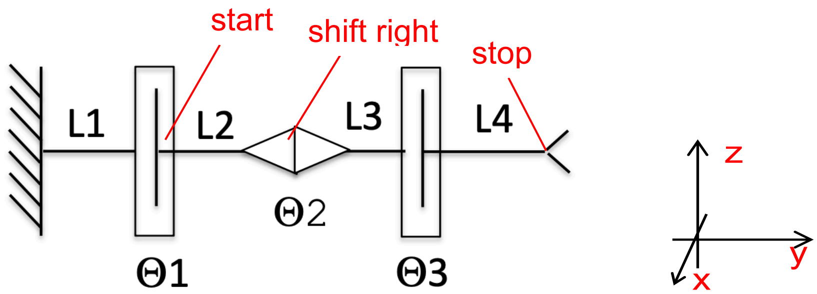
---

^1_2T =

---

^2_3T =

QUESTION 4 ***Velocity Propagation*** **(10 points)**



For the manipulator shown above:

- Set L1=0
- Use the DH-Notation

(a) Calculate all velocities

${}^{1}\omega_1$, ${}^{1}v_1$
${}^{2}\omega_2$, ${}^{2}v_2$
${}^{3}\omega_3$, ${}^{3}v_3$
${}^{4}\omega_4$, ${}^{4}v_4$

and write down the formula only for how to calculate ${}^{0}v_4$

(b) Build the Jacobian ${}^{3}J(\theta_1, \theta_2, \theta_3)$

QUESTION 5 *Reliability* (10 points)

Reliability of combinatorial systems

- Assume a system of *n* identical components *x* that all have the same reliability *r*.
- SER(..) denotes subassemblies in series (no redundancy)
- PAR(..) denotes subassemblies in parallel (full redundancy)

Note:

- $R(t) = e^{-\lambda t}$
- $MTTF = 1/\lambda$
- $MTTF = \int_0^\infty R(t)\,dt$

(a) Compute the reliability in terms of *r* for the component system:
SER(PAR(x, x, x, x), PAR(x, x))

(b) For a constant failure rate λ

Assume each component x has the failure rate $\lambda = 0.02$ failures per day.

- Calculate MTTF for a single component x.
- Calculate $R_X(t)$ at $t = 0$
- Calculate $R_X(t)$ at $t = MTTF$
- Plot the graph $R_X(t)$ in range [0, 100] days

(c) Calculate MTTF for these configurations

- Calculate MTTF for SER(x,x).
- Calculate MTTF for PAR(x,x).